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A game-theoretic rationale for EMU

by

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and

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Abstract

In this paper we show that from a macro-economic point of view the realisation of a European Monetary Union (EMU) may be a very rational decision of the participating countries. Despite the fact that countries loose their instrument for monetary policy we show, in a very general context, that in a non-cooperative world it may happen that welfare in each country increases by the introduction of an EMU. The analysis is performed in both a two and three country setting assuming that the economics of the involved countries can be described by some simple static reduced-form macro-economic models and that each country maximizes individual welfare.

Keywords: European Monetary Union, Welfare optimization, Nash equilibrium.

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1 Introduction

The rationale for an EMU is often critisized. Policymakers complain about the loss of a domestic monetary policy instrument and of the associated ability to adjust exchange rates. As Goodhart (1995) argues, the cost of losing the domestic monetary policy instrument will depend, in part, on the extent to which participating nations are likely to suffer asymmetric shocks and the extent to which fiscal policy can, and should, serve as an alternative to the use of monetary policy to foster adjustments.

In this paper we show in a very general game-theoretic framework that the decision of the loss of domestic monetary policy and come to an EMU may be very rational if one assumes a view of the world in which noncooperative policymaking of macroeconomic policies is more realistic than cooperative policymaking. For instance governments may negotiate in an international context about abandoning old (or designing new) rules, regulations or agreements if they believe it is profitable to do so. In our game-theoretic concept this simply implies that the noncooperative gains before the rule is implemented should be mutually less than the noncooperative gains after implementing. In this paper we will analyse an institutional change, such as the implementation of an EMU, in the same way. Therefore, the main aim of this paper is to provide new insights to the current literature of international cooperation, for a recent overview see Bryant (1995), in at least two ways:

(1) We give a new, as we believe, interpretation of the EMU-concept in stage three. We assume in an EMU that the objective of the independent EMU authorities is in one sense a cooperative one, i.e., its primary goal is to help the participating countries in realizing their objectives, but, due to its independent status, we assume that the game actually played between the participating countries and the EMU authorities is a noncooperative one.

(2) Under the assumption that the policy multipliers in the reduced form setup do not change after the introduction of an EMU we show that the realization of EMU depends crucially on the sign, size and the interrelationship of the various policy spillovers and the asymmetries among the participating economies.

To make our point in the simplest way, but there is no loss of generality in this choice, we start our analysis with a two-country model, each having a single policy target and two policy instruments. We assume that for each country the economy can be described by a reduced-form static expression which includes both domestic and foreign monetary and fiscal policy, and an exogenous term representing the rest-of-the-world variables.

Moreover, we consider for both countries a standard quadratic social loss function. Under a regime of insular policymaking, each policymaker takes the policies of the other as given and tries to reach its targets. This results in a Nash equilibrium strategy for their policies and corresponding social losses. This situation is compared with the case that both countries agree upon to form an EMU. That is, a situation in which both the economy and social loss functions of the countries are described by the same equations, but monetary policy is delegated to an independent EMU authority. Now, on the one hand the EMU authority sets monetary policy taking into account the welfare loss functions of the individual countries. On the other hand we model the independency of the EMU by assuming that it, like the individual countries, takes the (fiscal) policies of the individual countries as given and tries to reach its target given these fixed policies. Consequently, the equilibrium policy strategies are in this case obtained as the Nash solution of a three person game. For this situation, the corresponding welfare-loss function of each country is compared with the outcome in the non-EMU case.

Two specific situations are analyzed theoretically in more detail. The first case is that one country is relatively small compared to the other. We will show that in that case the large country will never gain by participating in an EMU. The second case studies two symmetric countries. One clear result which holds in this case is that if monetary policy is beggar-thy-neighbor and fiscal policy is locomotive, then both countries gain by participation in an EMU. Thus in that case the introduction of an EMU may be a rather rational decision. In a simulation we will show that in general the decision to come to an EMU is a rather ambiguous one which critically depends on the model parameters and on the existing asymmetries.

Since we found that in a two-country model, where one country is relatively small compared to the other, one can hardly expect that an EMU will be realised, the question arises what will happen if this model is extended by an additional large country. We analyse this question in a separate section. Intuitively one would expect that since two large countries are involved now, who may gain under some conditions from the introduction of a "bilateral" EMU, a small country joining these two in the EMU will in general be no problem for them. We will see that this intuition is correct, given that some parametric conditions are satisfied.

The organization of the paper is as follows. First, in section two we present the basic two-country model we will be dealing with in this paper. Then, in sections three and four we will analyze the model outcomes under a small country and symmetric country assumption, respectively. In section five we present some simulation results for the general model for a number of calibrated parameter choices. Section six treats the three-country model case and, finally, section seven contains some concluding remarks.

2 The two-country model

Consider two countries, which economies are represented by the following reduced-form model:

$$y = \alpha_1 g + \alpha_2 m + \alpha_3 g^* + \alpha_4 m^* + c \tag{1a}$$
$$y^* = \beta_1 g + \beta_2 m + \beta_3 g^* + \beta_4 m^* + c^* \tag{1b}$$

where each country is having a single policy target, y, which for example represents output and two policy instruments, g, the fiscal instrument and, m, the monetary instrument. In order to make a fair comparison with the EMU situation, as we will introduce further down, we will consider all variables in growth rates. c represents unmodeled phenomena including the influence of the rest-of-the world.
Next, assume that the social welfare loss functions are represented by

$$J = (y - \bar{y})^2 + g^2 + m^2 \tag{2a}$$

$$J^* = (y^* - \bar{y}^*)^2 + g^{*^2} + m^{*^2} \tag{2b}$$

Here $\bar{y}$ and $\bar{y}^*$ can be viewed as a target for domestic and foreign output, respectively. Note that by including the unmodeled phenomena c and c^* into these targets we may assume, without loss of generality, that $c = c^* = 0$ in (1).
Model-formulations in the literature concentrate mostly on frameworks with only one policy instrument and one (or more) policy targets. For example skipping the fiscal policy instrument in our formulation would resemble the model used by Canzoneri and Gray (1985) in their study of monetary policy games. The same stylized models are used by Ghosh and Masson (1994) and Hughes Hallett (1993). The latter shows that this reduced-form structure also fits in the framework commonly used in the central bank literature, such as Giavazzi and Pagano (1988) and Rogoff (1985). In comparison with theoretical central bank studies which focus primarily on monetary policy (see e.g. Broadbent and Barro (1995) en Morales and Padilla (1994)) the inclusion of a fiscal policy instruments is of major importance here. Not only is it clear that there is an intimate connection between monetary and fiscal policy in the sense that each policy must be analyzed and chosen with the other in mind (see Leeper (1993)) but also it shows to which extent fiscal policy will change in an EMU context.

In the literature there is some ambiguity about the inclusion of policy instruments in the welfare loss functions. For example Hughes Hallett (1993) includes policy instruments in the welfare loss functions whereas many others do not. One reason may be that the normalization of the desired targets for fiscal and monetary policy in both countries towards zero is no restriction and therefore g and m are sometimes also described as deviations from their desired paths. In our model, containing a single target and two instruments, the inclusion of policy instruments in the welfare loss functions is useful since it yields unique non-cooperative strategies. We show this aspect in appendix 1. Including some weights in the welfare loss functions for fiscal and monetary policy would yield a somewhat more general framework. This, however, at the expense of introducing more parameters which intricates the analytical tractability and it does not yield much more insight in most of the points we want to show.

As already indicated in the introduction, we assume that both countries are just interested in the minimization of their welfare-loss function and they set their policy instruments, i.e. fiscal and monetary policy, independent of eachother. This Nash equilibrium gives rise to the following unique policies, provided that the parameter det (see (4)) differs from zero (see appendix 1):

$$g_2^e = \frac{\alpha_1}{\det}\left[(1+\beta_3^2+\beta_4^2)\bar{y} - (\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*\right] \tag{3a}$$

$$m_2^e = \frac{\alpha_2}{\det}\left[(1+\beta_3^2+\beta_4^2)\bar{y} - (\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*\right] \tag{3b}$$

$$g_2^{*^e} = \frac{\beta_3}{\det}\left[(1+\alpha_1^2+\alpha_2^2)\bar{y}^* - (\alpha_1\beta_1+\alpha_2\beta_2)\bar{y}\right] \tag{3c}$$

$$m_2^{*^e} = \frac{\beta_4}{\det}\left[(1+\alpha_1^2+\alpha_2^2)\bar{y}^* - (\alpha_1\beta_1+\alpha_2\beta_2)\bar{y}\right] \tag{3d}$$

where

$$\det := (1+\alpha_1^2+\alpha_2^2)(1+\beta_3^2+\beta_4^2) - (\alpha_1\beta_1+\alpha_2\beta_2)(\alpha_3\beta_3+\alpha_4\beta_4). \tag{4}$$

Note that from these formulas it follows in particular that the equilibrium strategies satisfy the relationship $\alpha_1 m_2^e = \alpha_2 g_2^e$ and $\beta_3 m_2^{*^e} = \beta_4 g_2^{*^e}$, respectively. Substitution of these equilibrium strategies into the welfare-loss functions (2) yields then the following equilibrium values (see again appendix 1), respectively:

$$J^e = \frac{1+\alpha_1^2+\alpha_2^2}{\det^2}\left[(1+\beta_3^2+\beta_4^2)\bar{y} - (\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*\right]^2 \tag{5a}$$

or, equivalently (provided $\alpha_1 \neq 0$), $J^e = \frac{1+\alpha_1^2+\alpha_2^2}{\alpha_1^2} g_2^{e^2}$

$$J^{*^e} = \frac{1+\beta_3^2+\beta_4^2}{\det^2}\left[(1+\alpha_1^2+\alpha_2^2)\bar{y}^* - (\alpha_1\beta_1+\alpha_2\beta_2)\bar{y}\right]^2 \tag{5b}$$

or, equivalently (provided $\beta_3 \neq 0$), $J^{*^e} = \frac{1+\beta_3^2+\beta_4^2}{\beta_3^2} g_2^{*^{e^2}}$.

Next, consider the situation that both countries agree on an institutional change and form an EMU. That is, they agree that henceforth de facto both countries use the same currency and that monetary policy is determined by the independent EMU authority. Since in the EMU situation we have one growth rate for money we may replace m and m^* by the new monetary instrument m'. When setting this monetary instrument we assume that the EMU authority strives for minimizing a weighted sum of the welfare-loss functions of both countries. This gives rise to the following model:
The economies (1) of both countries are described under this institutional change by:

$$y = \alpha_1 g + (\alpha_2+\alpha_4)m' + \alpha_3 g^* \tag{6a}$$
$$y^* = \beta_1 g + (\beta_2+\beta_4)m' + \beta_3 g^* \tag{6b}$$

where m' is the monetary instrument determined by the EMU authority. Remark that we implicitly assume that this institutional change does not affect the reduced form policy multipliers. This is a somewhat heroic assumption since an institutional change is likely to alter the impact of these multipliers as well. For our game-theoretical analysis, however, this assumption is the most plausible one since this enables us to make a fair comparison between the two regimes. The welfare-loss functions for both countries are again given by (2) and the welfare-loss function the EMU likes to minimize is given by

$$wJ + (1-w)J^*, \tag{7}$$

where $0 \leq w \leq 1$ is a weight parameter which is set by the EMU-authority. Hence, in this formulation the loss of individual monetary policy is, partly, compensated by the "coordination" instrument w of the EMU-authority.

Note that the assumptions on the welfare-loss functions are crucial for the rest of our analysis. By our first assumption that these loss functions remain the same for both countries under the “EMU regime”, we express the belief that on the one hand feelings regarding welfare-loss of people is indifferent for the fact who is responsible for setting policy instruments, and on the other hand that costs involved of maintaining EMU weigh out the costs involved of performing an own monetary policy. Our second assumption expresses the belief that the EMU as an independent authority has no individual goals, that is, we assume that for performing monetary policy it only takes into consideration the interests of the participating countries. As mentioned in the introduction, we consider a Nash equilibrium among the three players. However, within this Nash equilibrium there is some room for “coordination” by the EMU authority which is modelled by weight parameter w. In this context we deliberately do not speak about a central bank which primary focus is price stability. This remains an interesting topic for future research where one may consider a multi-target framework in which tradeoffs between inflation and growth have to be studied. Independence in our framework is related to the “coordination” parameter w. We assume that it will be set by the independent EMU-authority, but one could also imagine situations where w is determined by a bargaining process between the Member States.

In an equilibrium situation this yields then the following unique policies (see appendix 2) provided the regularity condition, that, the parameter $\det_2$ (see (9) below) differs from zero, is satisfied:

$$\begin{aligned} g^e_{\mathrm{EMU},2} &= \frac{\alpha_1}{\det_2}[(1+(1-w)(\beta_2+\beta_4)^2+\beta_3^2)\bar{y} - \\ &\quad ((1-w)(\alpha_2+\alpha_4)(\beta_2+\beta_4)+\alpha_3\beta_3)\bar{y}^*] \end{aligned} \tag{8a}$$

$$\begin{aligned} m'^e_{\mathrm{EMU},2} &= \frac{1}{\det_2}[(w(\alpha_2+\alpha_4)(1+\beta_3^2)-(1-w)\alpha_1\beta_1(\beta_2+\beta_4))\bar{y} + \\ &\quad +((1-w)(1+\alpha_1^2)(\beta_2+\beta_4)-w(\alpha_2+\alpha_4)\alpha_3\beta_3)\bar{y}^*] \end{aligned} \tag{8b}$$

$$g^{*e}_{\mathrm{EMU},2} = \frac{\beta_3}{\det_2}[(1+w(\alpha_2+\alpha_4)^2+\alpha_1^2)\bar{y}^* - (w(\alpha_2+\alpha_4)(\beta_2+\beta_4)+\alpha_1\beta_1)\bar{y}] \tag{8c}$$

where

$$\begin{aligned} \det_2 &:= (1+\alpha_1^2+w(\alpha_2+\alpha_4)^2)(1+\beta_3^2+(1-w)(\beta_2+\beta_4)^2) - \\ &\quad (\alpha_1\beta_1+w(\alpha_2+\alpha_4)(\beta_2+\beta_4))(\alpha_3\beta_3+(1-w)(\alpha_2+\alpha_4)(\beta_2+\beta_4)) \end{aligned} \tag{9}$$

Note that both in the non-EMU and the EMU case the equilibrium strategies of both countries do not take into account the direct effect their policies have on the other country. That is, in as well e.g. (3a,b) as (8a) we have that the fiscal instrument (and monetary instrument in the non-EMU case) is only influenced by the model parameter β_1 (and β_2 in the non-EMU case) via the determinant, which is a factor that equally influences all equilibrium strategies. Moreover, we observe that the equilibrium strategies (8) in the EMU-case depend in an inverse way on the weight the EMU-authority attaches to their welfare-loss function. That is, if the EMU-authority attaches a weight w to the welfare-loss function of the domestic country then e.g., apart from the determinant (see remark above), the parameter $\beta_2 + \beta_4$ only occurs with a weight $1 - w$ in (8a).
Substitution of these equilibrium strategies (8) into the welfare-loss functions (2) yields then the equilibrium outcomes (see appendix 2):

$$\begin{aligned} J^e_{\text{EMU}} &= \frac{1}{\det_2^2}\Big\{(1+\alpha_1^2)[(1+(1-w)(\beta_2+\beta_4)^2+\beta_3^2)\bar{y}- \\ &\quad ((1-w)(\alpha_2+\alpha_4)(\beta_2+\beta_4)+\alpha_3\beta_3)\bar{y}^*]^2 + \\ &\quad [(w(\alpha_2+\alpha_4)(1+\beta_3^2)-(1-w)\alpha_1\beta_1(\beta_2+\beta_4))\bar{y} + \\ &\quad ((1-w)(1+\alpha_1^2)(\beta_2+\beta_4)-w(\alpha_2+\alpha_4)\alpha_3\beta_3)\bar{y}^*]^2\ \Big\} \end{aligned} \tag{10a}$$

or, equivalenty (provided $\alpha_1 \neq 0$), $J^e_{\text{EMU}} = \left(\frac{1}{\alpha_1^2}+1\right) g^{e^2}_{\text{EMU},2} + m'^{e^2}_{\text{EMU},2}$, and

$$\begin{aligned} J^{*^e}_{\text{EMU}} &= \frac{1}{\det_2^2}\Big\{(1+\beta_3^2)[(\alpha_1\beta_1+w(\alpha_2+\alpha_4)(\beta_2+\beta_4))\bar{y}- \\ &\quad (1+\alpha_1^2+w(\alpha_2+\alpha_4)^2)\bar{y}^*]^2 + \\ &\quad [(w(\alpha_2+\alpha_4)(1+\beta_3^2)-(1-w)\alpha_1\beta_1(\beta_2+\beta_4))\bar{y} + \\ &\quad ((1-w)(1+\alpha_1^2)(\beta_2+\beta_4)-w(\alpha_2+\alpha_4)\alpha_3\beta_3)\bar{y}^*]^2\ \Big\} \end{aligned} \tag{10b}$$

or, equivalently (provided $\beta_3 \neq 0$), $J^{*^e}_{\text{EMU}} = \left(\frac{1}{\beta_3^2}+1\right) g^{*^{e2}}_{\text{EMU},2} + m'^{e^2}_{\text{EMU},2}$.
Now, consider equations (5) and (10).
In the sequel we will denote the difference between the equilibrium outcomes J^e_{EMU} and J^e by ΔJ, i.e. $\Delta J := J^e_{\text{EMU}} - J^e$, and similarly $J^{*^e}_{\text{EMU}} - J^{*^e}$ by ΔJ^*.
The following definition makes now sense:

Definition 1:

We call an EMU between both countries realizable for a given parameter set α_i, β_i

$(i = 1, \ldots, 4)$, w and target values $\bar{y}$ and $\bar{y}^*$ if both ΔJ and ΔJ^* are nonpositive. $\square$

In other words, we call the EMU realizable if both countries would not lose (in terms of welfare) from its introduction. The rest of the paper will be dealing with the question under which conditions on the parameters and targets EMU is realizable. As announced in the introduction, to deal with this question we take two approaches. First we will make some simplifying assumptions on the parameters, which enhance an analytic treatment of this problem. Second, we will perform a simulation study. That is, we will calculate for a number of "realistic" parameters and targets whether the EMU is realizable.

3 Realizability of EMU under a "small country" assumption

In this section we assume that the home country is relatively small compared to the foreign country. This is expressed by taking $\beta_1 = \beta_2 = 0$ in both (1b) and (6b). Substitution of these parameter values into (5) and (10) yields then the following equilibrium welfare-loss functions for the non-EMU and EMU case, respectively:

$$J^e := \frac{1+\alpha_1^2+\alpha_2^2}{\det^2}[(1+\beta_3^2+\beta_4^2)\bar{y} - (\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*]^2$$

$$J^{*^e} := \frac{1+\beta_3^2+\beta_4^2}{\det^2}[(1+\alpha_1^2+\alpha_2^2)\bar{y}^*]^2$$

$$\begin{aligned} J^e_{\mathrm{EMU}} &:= \frac{1}{\det_2^2}\{(1+\alpha_1^2)[(1+(1-w)\beta_4^2+\beta_3^2)\bar{y} - ((1-w)\alpha_5\beta_4+\alpha_3\beta_3)\bar{y}^*]^2 + \\ &\quad [w\alpha_5(1+\beta_3^2)\bar{y} + ((1-w)(1+\alpha_1^2)\beta_4 - w\alpha_5\alpha_3\beta_3)\bar{y}^*]^2\} \end{aligned}$$

$$\begin{aligned} J^{*^e}_{\mathrm{EMU}} &:= \frac{1}{\det_2^2}\{(1+\beta_3^2)[w\alpha_5\beta_4\bar{y} - (1+\alpha_1^2+w\alpha_5^2)\bar{y}^*]^2 + \\ &\quad [w\alpha_5(1+\beta_3^2)\bar{y} + ((1-w)(1+\alpha_1^2)\beta_4 - w\alpha_5\alpha_3\beta_3)\bar{y}^*]^2\}, \end{aligned}$$

where

$$\det := (1+\alpha_1^2+\alpha_2^2)(1+\beta_3^2+\beta_4^2) > 0$$
$$\det_2 := (1+\alpha_1^2+w\alpha_5^2)(1+\beta_3^2+(1-w)\beta_4^2) - w\alpha_5\beta_4(\alpha_3\beta_3+(1-w)\alpha_5\beta_4),$$

and we used α_5 to denote $\alpha_2+\alpha_4$.

Note from det and $\det_2$, respectively, that for the non-EMU case always an equilibrium exists, whereas this need not to be the case for the EMU situation. Now, under the assumption that for both situations an equilibrium occurs we have that

$$\begin{aligned} \Delta J^* &= \frac{1}{\det_2^2\det^2}\{\det^2\{(1+\beta_3^2)[w\alpha_5\beta_4\bar{y} - (1+\alpha_1^2+w\alpha_5^2)\bar{y}^*]^2 + \\ &\quad [w\alpha_5(1+\beta_3^2)\bar{y} + ((1-w)(1+\alpha_1^2)\beta_4 - w\alpha_5\alpha_3\beta_3)\bar{y}^*]^2\} \\ &\quad -\det_2^2(1+\beta_3^2+\beta_4^2)(1+\alpha_1^2+\alpha_2^2)^2\bar{y}^{*^2}\} \\ &= \frac{1}{\det_2^2(1+\beta_3^2+\beta_4^2)}w^2(1+\beta_3^2)[(1+\beta_3^2+\beta_4^2)\alpha_5\bar{y} - \\ &\quad ((1+\alpha_1^2+\alpha_5^2)\beta_4+\alpha_5\alpha_3\beta_3)\bar{y}^*]^2 \end{aligned} \tag{11}$$

The above equality can e.g. be obtained by spelling out both sides of the equality and comparing terms. This is a straightforward, though rather lengthy job, that gives not much insight and which we therefore omit.

From (11), we immediately see that $\Delta J^* \geq 0$. This observation leads directly to the conclusion:

Theorem 2:
Under a small country assumption almost never an EMU will be realized. This, since the large country is almost always confronted with a welfare-loss if an EMU is established.

Note that the only case under which an EMU might be realized is if the last expression in (11) between square brackets is zero. In that case we would have $\Delta J = 0$.
The example is an extreme case and, therefore, the intuition behind the result is obvious. Since in the non-EMU case the foreign (strong) country is not interdependent with the home (weak) country it can follow an independent policy which guarantees the strong country the smallest possible welfare loss. Hence, in the EMU-case interdependence is accomplished through the EMU-authority which may decrease welfare for the stronger country.
One could also argue that in this context the assumption that the players play a Nash-equilibrium is maybe not the most realistic one, and that a Stackelberg-equilibrium may be a more appropriate one. This is, however, beyond the scope of this paper and left as a problem for future research.

4 Realizability of EMU under a "symmetric country" assumption

We now consider the case that the economies of both countries are similar. This is expressed by taking $\beta_1 = \alpha_3$, $\beta_2 = \alpha_4$, $\beta_3 = \alpha_1$, $\beta_4 = \alpha_2$ and $\bar{y} = \bar{y}^*$ in (1) and (6). For analytical purposes we will, moreover, assume that $\alpha_3 = \theta_1\alpha_1$ and $\alpha_4 = \theta_2\alpha_2$. Note that in economic models usually $|\theta_i| \leq 1$, $i = 1, 2$. We will make therefore this assumption throughout the rest of this section.
This results in the following model for the economies in the non-EMU situation:

$$\begin{aligned} y &= \alpha_1 g + \alpha_2 m + \theta_1\alpha_1 g^* + \theta_2\alpha_2 m^* \\ y^* &= \theta_1\alpha_1 g + \theta_2\alpha_2 m + \alpha_1 g^* + \alpha_2 m^* \end{aligned} \tag{12}$$

From (5) we have that the corresponding equilibrium welfare loss functions are:

$$J^e = J^{*^e} := \frac{1 + \alpha_1^2 + \alpha_2^2}{\det^2}[(1 + (1 - \theta_1)\alpha_1^2 - (1 - \theta_2)\alpha_2^2]^2\bar{y}^2 \tag{13}$$

where $\det := (1 + \alpha_1^2 + \alpha_2^2)^2 - (\theta_1\alpha_1^2 + \theta_2\alpha_2^2)^2 > 0$.

Note that, consequently, we can rewrite $J^e = J^{*^e} = \frac{1+\alpha_1^2+\alpha_2^2}{(1+(1+\theta_1)\alpha_1^2+(1+\theta_2)\alpha_2^2)^2}\bar{y}^2$.
Similarly, we obtain for the EMU case:

$$\begin{aligned} y &= \alpha_1 g + \alpha_2(1 + \theta_2)m' + \theta_1\alpha_1 g^* \\ y^* &= \theta_1\alpha_1 g + \alpha_2(1 + \theta_2)m' + \alpha_1 g^* \end{aligned}$$

and

$$J^e_{\text{EMU}} = J^{*^e}_{\text{EMU}} := \frac{1}{\det_2^2}(1 + \alpha_1^2 + (1 + \theta_2)^2\alpha_2^2)(1 + (1 - \theta_1)\alpha_1^2)^2\bar{y}^2$$

where,

$$\begin{aligned} \det{}_2 &:= (1 + \alpha_1^2)^2 + (1 + (1 - \theta_1)\alpha_1^2)\alpha_2^2(1 + \theta_2)^2 - \theta_1^2\alpha_1^4 \\ &= (1 + (1 - \theta_1)\alpha_1^2)(1 + (1 + \theta_1)\alpha_1^2 + (1 + \theta_2)^2\alpha_2^2) > 0. \end{aligned}$$

Note from these expressions that we can rewrite the welfare-loss functions as

$$J_{\mathrm{EMU}}^{e} = J_{\mathrm{EMU}}^{*^{e}} = \frac{1+\alpha_1^2+(1+\theta_2)^2\alpha_2^2}{(1+(1+\theta_1)\alpha_1^2+(1+\theta_2)^2\alpha_2^2)^2}\bar{y}^2. \tag{14}$$

Furthermore, note that due to the symmetry assumption the equilibrium welfare-loss functions in the EMU case become independent of w. From these expressions we obtain the following result:

Theorem 3:
Under a "symmetric country" assumption and the additional assumptions that an expansionary monetary policy has a negative influence on foreign output (i.e. $\theta_2 < 0$) and that an expansionary fiscal policy has positive effect on foreign output (i.e. $\theta_1 > 0$), an EMU is realizable.

Intuitively, this result makes sense. Since in a situation where $\theta_2 < 0$, a monetary expansion is a beggar-thy-neighbour policy, and where $\theta_1 > 0$, a fiscal expansion is a locomotive policy, it is better to let an independent EMU authority, who considers the interests of both countries, decide on the impact of the beggar-they-neighbour policies. The justification for this rationale follows by rewriting ΔJ $(= \Delta J^*)$ as follows:

$$\begin{aligned}
\Delta J &= \frac{\bar{y}^2}{\det^2 \det_2^2}\{\det{}^2(1+\alpha_1^2+(1+\theta_2)^2\alpha_2^2)(1+(1-\theta_1)\alpha_1^2)^2 - \\
&\quad \det{}_2^2(1+\alpha_1^2+\alpha_2^2)(1+(1-\theta_1)\alpha_1^2+(1-\theta_2)\alpha_2^2)^2\} \\
&= \theta_2\frac{\bar{y}^2}{\det^2 \det_2^2}\alpha_2^2((1-\theta_1)\alpha_1^2+1)^2((1-\theta_1)\alpha_1^2+(1-\theta_2)\alpha_2^2+1)^2[\\
&\quad (\theta_1+1)((\theta_1-1)\theta_2+2\theta_1)\alpha_1^4+((1+\theta_2)^2(2\theta_1-\theta_2)\alpha_2^2+2(\theta_1-\theta_2))\alpha_1^2 \\
&\quad -\theta_2((\theta_2+1)^2\alpha_2^2+1)],
\end{aligned} \tag{15}$$

where the last equality can be verified by a lengthy comparison of terms on both sides of the equality sign. Introducing

$$\begin{aligned}
s &:= [(\theta_1+1)((\theta_1-1)\theta_2+2\theta_1)\alpha_1^4+((1+\theta_2)^2(2\theta_1-\theta_2)\alpha_2^2+ \\
&\quad 2(\theta_1-\theta_2))\alpha_1^2-\theta_2((\theta_2+1)^2\alpha_2^2+1)],
\end{aligned} \tag{16}$$

we see from (15) that the sign of ΔJ equals minus the sign of s. Given, furthermore, that $\theta_1 > 0$ and $\theta_2 < 0$, we observe immediately that the sign of ΔJ is negative, which proves theorem 3.

5 A simulation study

For a better understanding of the theoretical results we perform in this section a simulation analysis. We will concentrate on the analysis of the realizability of EMU in a "symmetric country" model on which we will perform some asymmetric shocks. As we have already noticed, the outcome of the game depends strongly on the parameter choices determining the spillovers in the reduced form model. In the literature for multi-country studies there is still a debate about the direction and strength of international spillovers. For example, Bryant et al. (1990) and Frankel(1995) show, by using a range of macroeconomic world-models, that there is little empirical consensus on the size and the sign of various spillovers. Also Whitley (1992) finds that in models describing the European economy spillover effects to other European economies, originating from a single-country European expansion, are negligible. Douven and Plasmans (1996) express their beliefs that these findings are merely a result of the modelling strategies used and that in practice the size of these spillovers may be considerably larger than advocated by these models. In order to cope with these different findings, we will consider a wide range of possible parameter choices for the various spillovers and thus leave this debate undecided.

Another important aspect for our conclusions is the "symmetric" assumption. As shown by Bayoumi and Eichengreen (1992) shocks across EU-regions can be rather asymmetric, therefore, we will present some examples how the two different regimes respond to asymmetric shocks.

Consider again the symmetric framework in (12). In order to keep the simulation tractable, we assume that $\alpha_1 = \alpha_2 = 1$, which reflects the fact that a government is indifferent in using a fiscal or monetary instrument for expanding output. The elasticity choice of 1 turns out to be just a scaling choice since we are only interested in the relative performance of the two regimes. Furthermore, we consider $-0.5 < \theta_i < 0.5,\ i = 1, 2$, which is likely to cover the wide range of empirical outcomes found by most multi-country studies. The choice for $\bar{y}, \bar{y}^*$ is arbitrary and the qualitative results are insensitive to the size of it, as long as we stay "symmetric". We started with a choice of 3, which reflects a 3% growth rate in output. We deviated from the "symmetric" assumption by simulating different values for $\bar{y}$. This difference may not only reflect a different desired value for output in the first country but may also reflect the fact that the two countries are hit by different external shocks (represented by c and c^* in the reduced-form model (1) in section two). This can be easily seen by bringing c to the left hand side in equation

(1a). Substitution of $y - c$ in the first term in (2a) yields then $(y - (\bar{y} + c))^2$. The simulation comparison results of the two regimes are presented in figure 1. The four diagrams in figure 1 represent different asymmetric shocks, using different values for c. Each symbol in the diagrams represents a certain choice for (θ_1, θ_2) and c. The symbols are characterised as follows:

- $+$ indicates that an EMU is realizable
- $*$ indicates that both countries are worse off under an EMU.
- $\times$ only country one (home) gains under an EMU.
- $\circ$ only country two (foreign) gains under an EMU.

Note that in our calculations for each pair of (θ_1, θ_2) we had to calculate the optimal choice for the negotiation parameter weight w.

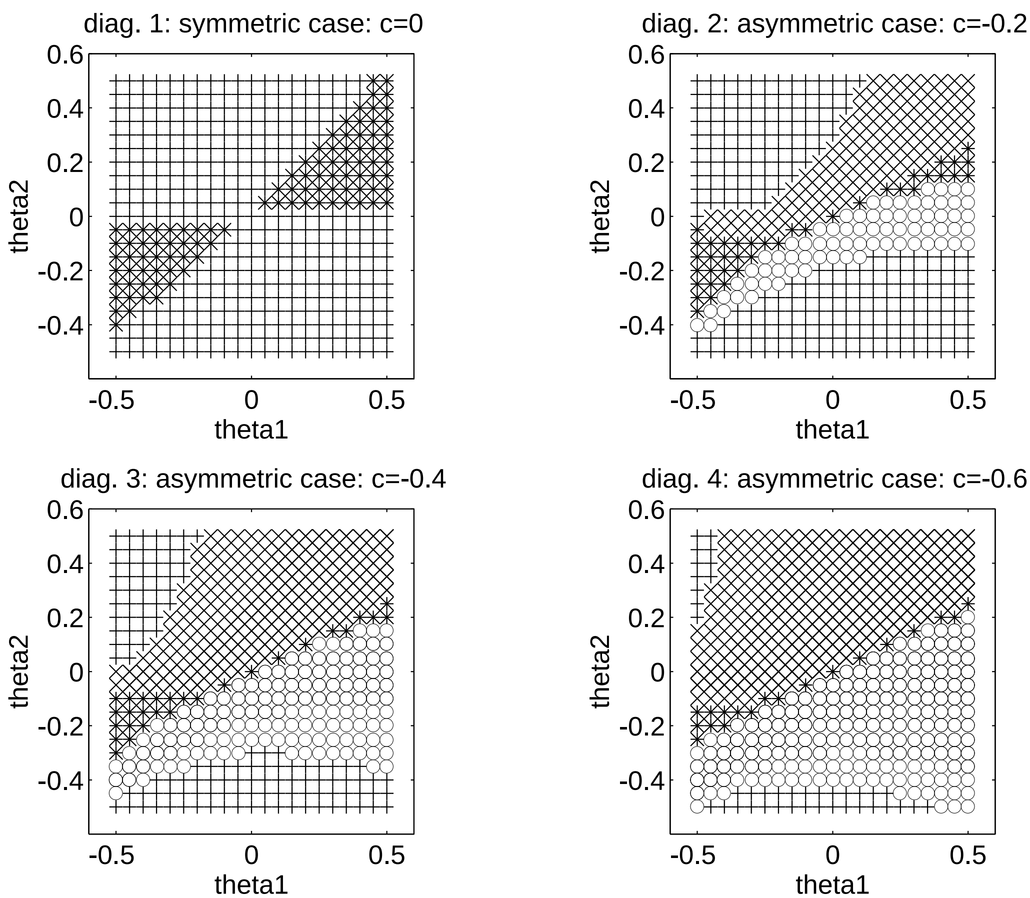

Figure 1: Asymmetric shocks under a "symmetric country" assumption

First we searched for values for which an EMU is realizable and if this was not found we searched for a value where at least one country was better off. Remark, that we did not find a situation in which an EMU was not realizable but there was a w for which only country one was better off and there was another w for which only country two was better off. The first diagram represents the symmetric case in which there is no shock and we see that for $\theta_1 > 0, \theta_2 < 0$ (a fiscal expansion is locomotive and a monetary expansion is beggar thy neighbour) that both countries are better off under an EMU. The mirror image ($\theta_2 < 0, \theta_1 > 0$) is also favourable but this depends on our particular choice $\alpha_1 = \alpha_2 = 1$. Under this assumption it easy to show that $s < 0$ in (16). Regarding EMU, unfavourable outcomes can be found in the area where the sign of the fiscal spillover effect equals the sign of the monetary spillover effect, but is larger in size. Diagrams 2-4 in figure 1 show that the symmetric assumption is an important one since a small external shock in one of the countries can already trouble the outcome. In diagrams 2-4, we only decreased the value of c, from zero to -0.2, -0.4 to -0.6, suggesting an increasing negative external shock in the home country. We observe that as the size of the shock (or differences in preferences) increases not only the EMU realizability area vanishes but the amount of $*$'s vanishes as well. In general, as one can see from the diagrams, in the asymmetric cases it depends on the size and sign of the spillovers and the shock implied, which country gains under an EMU and which country not.

6 The three-country model

In the stylized models of sections 3 and 4 we saw that the realizability of an EMU depends in particular on the size of both countries. If one of the countries is relatively small, an EMU will not be realized; if they are similar, the realizability depends on the specific model parameters.

Given this observation, the question arises what will happen in a three-country model where one country is relatively small and the other two are symmetric (in the sense of section 4). As we saw in section 3 in a two-country model the realization of an EMU is blocked, under a small country assumption, since the large country does not gain from its realization. Given this observation and the fact that in the symmetric case an EMU may be realizable, one would expect that in a three-country model the small country may be sucked in an EMU realization of the two symmetric countries. This observation is also made by Alesina and Grilli (1993) where they express some skepticism on the idea of a "multi-speed" union. They argue that once, for example in our model, the two large countries form a restricted union they will never agree to enlarge the union, on a later date, with a small country, whereas an EMU would be feasible at "one speed". To analyze this aspect we consider the following model:

The economies of the three countries in the non-EMU case are described by (see (12))

$$\begin{aligned} y &= \alpha_1 g + \alpha_2 m + \theta_1 \alpha_1 g^* + \theta_2 \alpha_2 m^* \\ y^* &= \theta_1 \alpha_1 g + \theta_2 \alpha_2 m + \alpha_1 g^* + \alpha_2 m^* \\ y_s &= \gamma_1 g + \gamma_2 m + \gamma_3 g^* + \gamma_4 m^* + \gamma_5 g_s + \gamma_6 m_s, \end{aligned} \tag{17}$$

where, y_s, g_s and m_s denote output, fiscal and monetary policy of the small country and we assume, to simplify the analyses, that both α_1 and γ_5 differ from zero. The welfare-loss functions are assumed to be given by (see (2)) J, J^* and $J_s := (y_s - \bar{y}_s)^2 + g_s^2 + m_s^2$, where $\bar{y}_s$ is the output target for the small country. Under the assumption, again, that every country is just interested in the minimization of its own welfare-loss function, we obtain in an equilibrium situation (under the regularity condition that the parameter det $\neq 0$, see below) the following strategies (see (3) and appendix 3):

$$g_3^e = g_3^{*^e} = \frac{\alpha_1}{\det}(1 + (1-\theta_1)\alpha_1^2 + (1-\theta_2)\alpha_2^2)\bar{y} \tag{18a}$$

$$m_3^e = m_3^{*^e} = \frac{\alpha_2}{\det}(1 + (1-\theta_1)\alpha_1^2 + (1-\theta_2)\alpha_2^2)\bar{y} \tag{18b}$$

where $\det = (1+\alpha_1^2+\alpha_2^2)^2 - (\theta_1\alpha_1^2+\theta_2\alpha_2^2)^2$,
and

$$g^e_{s,3} = \frac{\gamma_5}{d}[-((\gamma_1+\gamma_3)\alpha_1+(\gamma_2+\gamma_4)\alpha_2)\bar{y}+(1+(1+\theta_1)\alpha_1^2+(1+\theta_2)\alpha_2^2)\bar{y}_s] \quad (18c)$$

$$m^e_{s,3} = \frac{\gamma_6}{d}[-((\gamma_1+\gamma_3)\alpha_1+(\gamma_2+\gamma_4)\alpha_2)\bar{y}+(1+(1+\theta_1)\alpha_1^2+(1+\theta_2)\alpha_2^2)\bar{y}_s] \quad (18d)$$

From the above formulas it is immediately clear that the following relations hold: $d>0$, $\alpha_2 g^e_s = \alpha_1 m^e_3$ and $\gamma_6 g^e_{s,3} = \gamma_5 m^e_{s,3}$. This yields the equilibrium welfare-loss functions (see also (13)):

$$J^e = J^{*^e} = \frac{1+\alpha_1^2+\alpha_2^2}{\alpha_1^2} g_3^{e^2} \quad (19a)$$

and

$$J^e_s = \frac{1+\gamma_5^2+\gamma_6^2}{\gamma_5^2} g_{s,3}^{e^2} \quad (19b)$$

Next, consider the case that the three countries form an EMU and the EMU's goal is to minimize a weighted sum of the welfare-loss functions, i.e. $J_{\text{EMU}} := w_1 J + w_2 J^* + w_3 J_s$ with $w_1+w_2+w_3=1$. Introducing $\alpha_5 := \alpha_2(1+\theta_2)$ and $\gamma_7 := \gamma_2+\gamma_4+\gamma_6$ we obtain from (17) the following model equations:

$$\begin{aligned} y &= \alpha_1 g + \alpha_5 m' + \theta_1\alpha_1 g^* \\ y^* &= \theta_1\alpha_1 g + \alpha_5 m' + \alpha_1 g^* \\ y_s &= \gamma_1 g + \gamma_7 m' + \gamma_3 g^* + \gamma_5 g_s \end{aligned}$$

In appendix 3 it is shown that, provided $\det_3 \neq 0$ (see below), the equilibrium strategies are

$$\begin{aligned} g^e_{\text{EMU},3} = g^{*^e}_{\text{EMU},3} &= \frac{1}{\det_3}\alpha_1(1+(1-\theta_1)\alpha_1^2)[((1+\gamma_5^2)(1+w_3\gamma_7^2) \\ &\quad -w_3\gamma_1\gamma_5\gamma_7^2)\bar{y} - w_3\alpha_5\gamma_7\bar{y}_s] \end{aligned} \quad (20a)$$

$$m'^{e}_{\mathrm{EMU},3} = \frac{1}{det_3}(1+(1-\theta_1)\alpha_1^2)[(1+(1+\theta_1)\alpha_1^2)w_3\gamma_7\bar{y}_s - \\ ((1+\gamma_5^2)(w_3\alpha_1\gamma_3\gamma_7+(w_3-1)\alpha_5)+w_3\alpha_1\gamma_1\gamma_7(1-\gamma_3\gamma_5))\bar{y}] \tag{20b}$$

$$g^{e}_{s,\mathrm{EMU},3} = \frac{1}{det_3}(1+(1-\theta_1)\alpha_1^2)[((1+(1+\theta_1)\alpha_1^2)(\gamma_5+w_3\gamma_5\gamma_7^2 - \\ -w_3\gamma_1\gamma_7^2)+\alpha_5(\alpha_5\gamma_5(1-w_3)+w_3\alpha_1\gamma_3\gamma_7(\gamma_1-\gamma_5)))\bar{y}_s - \\ \gamma_1(\alpha_1\gamma_3+(1-w_3)\alpha_5\gamma_7-w_3\alpha_1\gamma_1\gamma_7^2+\alpha_1\gamma_5+w_3\alpha_1\gamma_5\gamma_7^2)\bar{y}] \tag{20c}$$

where

$$\mathrm{det}_3 = (1+(1-\theta_1)\alpha_1^2)[(1+(1+\theta_1)-\alpha_1^2)((1+\gamma_5^2)(w_3\gamma_7^2+1)-w_3\gamma_1\gamma_5\gamma_7^2) \\ -\alpha_5((1+\gamma_5^2)(w_3\alpha_1\gamma_3\gamma_7+(w_3-1)\alpha_5)+w_3\alpha_1\gamma_1\gamma_7(1-\gamma_3\gamma_5))],$$

and the corresponding welfare-loss functions:

$$J^{e}_{\mathrm{EMU}} = J^{*^{e}}_{\mathrm{EMU}} = \left(\frac{1}{\alpha_1^2}\right) g^{e^2}_{\mathrm{EMU},3} + m'^{e^2}_{\mathrm{EMU},3} \tag{21a}$$

and

$$J^{e}_{s,\mathrm{EMU}} = \left(\frac{1}{\gamma_5^2}+1\right) g^{e^2}_{s,\mathrm{EMU},3} + m'^{e^2}_{\mathrm{EMU},3} \tag{21b}$$

From (19a), (21a) we immediately observe that if the large countries have already reached their target output before "the game" starts, i.e. $\bar{y}=0$, then almost never an EMU will be realized if the small country has not yet reached its target output too, i.e. $\bar{y}_s \neq 0$. This, irrespective of the fact whether the parameter $\theta_2 < 0$. So, our first conclusion is that there exist situations where the inclusion of a small country into the EMU negotiation process may block the realization of an EMU between two symmetric countries.

On the other hand, the converse situation can also occur. The four player game is however rather complex. Therefore we will concentrate on some simulation results and discuss also the impact of the EMU authority through their "coordination" parameter w, since we believe that this is an interesting new aspect in these type of models.

The simulations in figure 2 represent the following parameter choices in (17). As in our

first simulation we used $\alpha_1 = \alpha_2 = 1$, $-0.5 < \theta_1, \theta_2 < 0.5$, $\bar{y} = \bar{y}^* = 3$. For the small country we have chosen the following values: $\bar{y_s} = 3$, $\gamma_1 = \gamma_3 = \theta_1\alpha_1$, $\gamma_2 = \gamma_4 = \theta_2\alpha_2$ and $\gamma_5 = \gamma_6 = 0.2$. As in the previous simulation every symbol in the figure represents a certain combination (θ_1, θ_2) and for each successive diagram in the figure we enlarged the coordination search of the EMU- authority. The symbols are characterised as follows:
- $+$ indicates that an EMU is realizable
- $*$ indicates that all three countries are worse off under an EMU.
- $\times$ indicates that (only) the small country gains under an EMU.
- $\circ$ indicates that (only) the two large countries gain under an EMU

Furthermore, the parameter weight, in the caption of each diagram, is described as follows: $w_1 = w_2 = \frac{1}{2}(1 - \frac{2}{3}\text{weight})$, $w_3 = \frac{2}{3}\text{weight}$. So, by enlarging for each successive diagram the w_1, w_2, w_3 space, the EMU authority may find better game outcomes. As one can see, sometimes the symbols $\times$, $\circ$ appear for the same (θ_1, θ_2) combination. for such a combination an "all three win" situation is not available and thus has the EMU-authority to choose for either the two large countries or the small country. As expected the $*$'s vanish and the number of $+$'s increases as the coordination space is enlarged. The main point we want to show is, however, that in general it is very hard to draw certain conclusions on questions like: "Who is better off?". In this example we find again some evidence that if the EMU authority picks out the right "coordination" policy an EMU seems most realizable in the $\theta_1 > 0, \theta_2 < 0$ area.

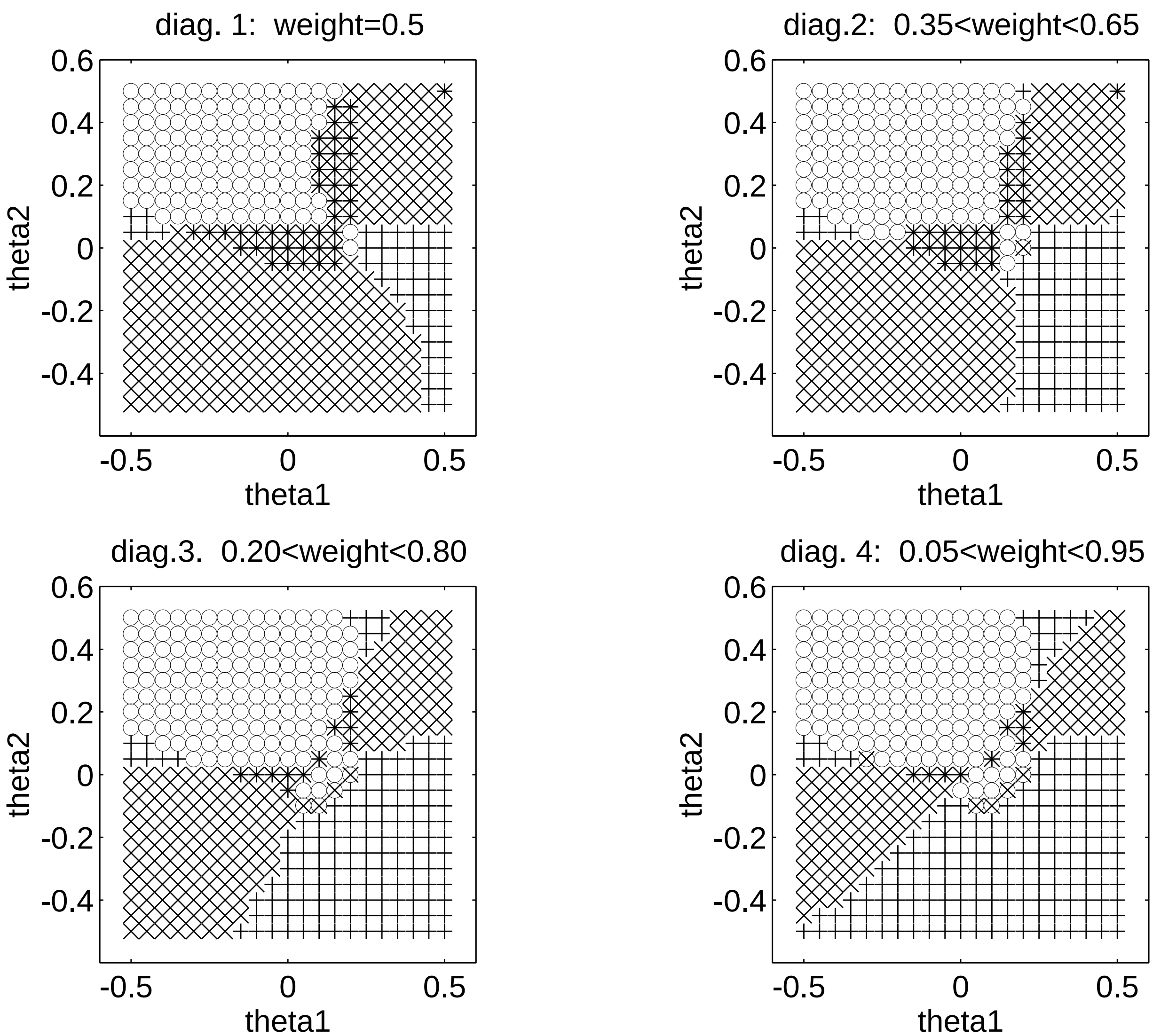

Figure 2: Enlarging the "coordination" space of the EMU-authority in a three country world

7 Concluding remarks

In this paper we showed in a theoretical framework that from a welfare optimization point of view the decision of countries to form an EMU may be very rational in the context of a noncooperative world. The analysis was based on an elementary reduced form macroeconomic model. In a two country model setting we derived analytic expressions for the optimal welfare functions of both countries in both the situations that they agree to form an EMU and not. In an EMU-situation we assume that the EMU-authority controls monetary policy and optimizes a weighted sum of the individual welfare functions of EU-countries but we assume that the actual game among the EU-countries and the EU-authority is played in a non-cooperative mode. We believe that this formulation makes more sense than cooperative frameworks where it is assumed that, in order to exploit international interdependencies positively, countries commit themselves to play an "unstable" Pareto efficient outcome. By comparing these welfare functions for the different regimes we saw that it may happen that in an EMU-regime welfare increases for both countries. We called an EMU realizable if this situation occurs. Two special cases were analyzed in more detail. First, we showed that under a small country assumption an EMU is almost never realizable. Second, we elaborated the problem under a symmetric country assumption. One observation we made under this assumption was that an EMU is realizable if monetary policy is beggar-thy-neighbour and fiscal policy is locomotive. To see how far these conclusions also hold in a more general multi-country model setting, we considered a three country model composed off two symmetric countries and one small country. Again, we derived analytic formulas for the optimal welfare functions of the three countries. In particular it turned out that the above mentioned observation we made in the two symmetric country model framework does not carry over to this new situation. We illustrated that the inclusion of a third small country into the symmetric country framework may have as well a positive as a negative effect on the realizability of an EMU. Concluding we could say that in our game-theoretic analysis we could not find much evidence that the loss of domestic monetary policy and moving to an EMU is associated with costs for all countries. On the contrary, in several examples we showed that even in a very simple reduced form framework the costs of moving to an EMU depend on the sign and size of the various spillovers and the existing asymmetries among the EU-economies.

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Appendix 1

In this appendix we calculate the Nash solution for the two country model (1), (2). The assumption that each country minimizes its own welfare-loss function w.r.t. its policy instruments g and m, taking the effect of the other country for granted, yields the following four conditions (obtained by differentiating (2a) w.r.t. g and m, respectively, and (2b) w.r.t. g^* and m^*, respectively):

$$(\alpha_1^2+1)g+\alpha_1\alpha_2 m+\alpha_1\alpha_3 g^*+\alpha_1\alpha_4 m^*=\alpha_1\bar{y} \tag{1.1a}$$
$$\alpha_1\alpha_2 g+(\alpha_2^2+1)m+\alpha_2\alpha_3 g^*+\alpha_2\alpha_4 m^*=\alpha_2\bar{y} \tag{1.1b}$$
$$\beta_1\beta_3 g+\beta_2\beta_3 m+(\beta_3^2+1)g^*+\beta_3\beta_4 m^*=\beta_3\bar{y}^* \tag{1.1c}$$
$$\beta_1\beta_4 g+\beta_2\beta_4 m+\beta_3\beta_4 g^*+(\beta_4^2+1)m^*=\beta_4\bar{y}^* \tag{1.1d}$$

So, there exists a unique equilibrium strategy if and only if this set of equations has a unique solution for g, m, g^* and m^*. Introducing

$$A:=\begin{pmatrix}\alpha_1^2+1 & \alpha_1\alpha_2 & \alpha_1\alpha_3 & \alpha_1\alpha_4\\ \alpha_1\alpha_2 & \alpha_2^2+1 & \alpha_2\alpha_3 & \alpha_2\alpha_4\\ \beta_1\beta_3 & \beta_2\beta_3 & \beta_3^2+1 & \beta_3\beta_4\\ \beta_1\beta_4 & \beta_2\beta_4 & \beta_3\beta_4 & \beta_4^2+1\end{pmatrix}$$

$$\underline{b}:=\begin{pmatrix}\alpha_1\bar{y}\\ \alpha_2\bar{y}\\ \beta_3\bar{y}^*\\ \beta_4\bar{y}^*\end{pmatrix} \text{ and } \underline{u}:=\begin{pmatrix}g\\ m\\ g^*\\ m^*\end{pmatrix},$$

we see that (1.1) can be rewritten as $A\underline{u}=\underline{b}$.

So a unique equilibrium exists if and only if the determinant of A, which we will denote by det, differs from zero. The equilibrium strategies are then $\underline{u}^e := A^{-1}\underline{b}$, where A^{-1} denotes the inverse of matrix A.

Some straightforward, though lengthy, calculations show that

$$\det=(1+\alpha_1^2+\alpha_2^2)(1+\beta_3^2+\beta_4^2)-(\alpha_1\beta_1+\alpha_2\beta_2)(\alpha_3\beta_3+\alpha_4\beta_4) \tag{1.2}$$

which yields the uniqueness result as advertised in section 2. Note, that if we had skipped the monetary and fiscal policy parameters, m and g, in both welfare functions then the

determinant of A would be zero. So, in that case either no equilibrium strategy exists or there exists more than one equilibrium strategy. .
Next, we show that

$$\underline{u}^e := \begin{pmatrix} g^e \\ m^e \\ g^{*^e} \\ m^{*^e} \end{pmatrix} := \frac{1}{\det} \begin{pmatrix} \alpha_1(1+\beta_3^2+\beta_4^2)\bar{y} - \alpha_1(\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^* \\ \alpha_2(1+\beta_3^2+\beta_4^2)\bar{y} - \alpha_2(\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^* \\ \beta_3(1+\alpha_1^2+\alpha_2^2)\bar{y}^* - \beta_3(\alpha_1\beta_1+\alpha_2\beta_2)\bar{y} \\ \beta_4(1+\alpha_1^2+\alpha_2^2)\bar{y}^* - \beta_4(\alpha_1\beta_1+\alpha_2\beta_2)\bar{y} \end{pmatrix}$$

satisfies (1.1a). Since in a similar way it can be shown that also (1.1b-1.1d) hold, we will omit that part of the proof.
To prove that $\underline{u}^e$ satisfies (1.1a) we first note that m^e and g^e satisfy $\alpha_1 m^e = \alpha_2 g^e$. Next, consider the lefthandside of equation (1.1a). From this we have that

$$\begin{aligned}
&\det\{(\alpha_1^2+1)g^e + \alpha_1\alpha_2 m^e + \alpha_1\alpha_3 g^{*^e} + \alpha_1\alpha_4 m^{*^e}\} = \\
&\det\{(1+\alpha_1^2+\alpha_2^2)g^e + \alpha_1(\alpha_3 g^{*^e} + \alpha_4 m^{*^e})\} = \\
&(1+\alpha_1^2+\alpha_2^2)[\alpha_1(1+\beta_3^2+\beta_4^2)\bar{y} - \alpha_1(\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*] + \\
&\alpha_1(\alpha_3\beta_3+\alpha_4\beta_4)[(1+\alpha_1^2+\alpha_2^2)\bar{y}^* - (\alpha_1\beta_1+\alpha_2\beta_2)\bar{y}] = \\
&\alpha_1[(1+\alpha_1^2+\alpha_2^2)(1+\beta_3^2+\beta_4^2) - (\alpha_1\beta_1+\alpha_2\beta_2)(\alpha_3\beta_3+\alpha_4\beta_4)]\bar{y} = \alpha_1 \cdot \det \bar{y},
\end{aligned}$$

which shows the claim (since by assumption $\det \neq 0$).
Finally, we consider the corresponding welfare-loss functions. We will only show the correctness of the formula for J^e, the correctness of the formula for J^{*^e} is proved in a similar way and is, therefore, skipped.
Note that, by substituting $\underline{u}^e$ into (2a), J^e can be rewritten as:

$$J^e = ((\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4)\underline{u}^e - \bar{y})^2 + g^{e^2} + m^{e^2}.$$

Using the above formulas for $\underline{u}^e$, we get

$$\begin{aligned}
J^e \ &= \ \frac{1}{\det^2}\{((\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4)\underline{u}^e \cdot \det\ - \bar{y}\cdot \det)^2 + \\
&\quad (\alpha_1^2+\alpha_2^2)[(1+\beta_3^2+\beta_4^2)\bar{y} - (\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*]^2\}.
\end{aligned}$$

Simple calculations show that

$$(\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4)\underline{u}^e \cdot \det\ - \bar{y} \cdot \det = -(1+\beta_3^2+\beta_4^2)\bar{y} + (\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*.$$

Substitution of this equality into J^e then gives

$$J^e = \frac{1}{\det^2}(1+\alpha_1^2+\alpha_2^2)[(1+\beta_3^2+\beta_4^2)\bar{y} - (\alpha_3\beta_3+\alpha_4\beta_4)\bar{y}^*]^2,$$

which completes the proof.

Another, more elegant proof, in case $\alpha_1 \neq 0$ of the above expression for J^e is obtained by noting that

$$(\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4)\underline{u}^e \cdot \det\ - \bar{y} \cdot \det = \frac{1}{\alpha_1}[\alpha_1(\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4)\underline{u}^e \cdot \det\ - \alpha_1\bar{y} \cdot \det]$$
$$= \frac{1}{\alpha_1}[\alpha_1^2 g^e + \alpha_1\alpha_2 m^e + \alpha_1\alpha_3 g^{*^e} + \alpha_1\alpha_4 m^{*^e} - \alpha_1\bar{y}]\det,$$

which can be rewritten, using (1.1a), as $-\frac{1}{\alpha_1}g^e \det$.
This obervation gives now immediately that

$$\begin{aligned} J^e &= \frac{1}{\alpha_1^2}g^{e^2} + g^{e^2} + m^{e^2} \\ &= \frac{1}{\alpha_1^2}(g^{e^2} + \alpha_1^2 g^{e^2} + \alpha_1^2 m^{e^2}) \\ &= \frac{1}{\alpha_1^2}(g^{e^2} + \alpha_1^2 g^{e^2} + \alpha_2^2 g^{e^2}) \\ &= \frac{1}{\alpha_1^2}(1+\alpha_1^2+\alpha_2^2)g^{e^2}. \end{aligned}$$

Appendix 2

In this appendix we calculate the equilibrium strategies and corresponding welfare-loss functions if the two countries agree to form an EMU. The both necessary and sufficient conditions for the existence of a unique equilibrium strategy are, in this case, that the following set of three equations has a unique solution for g, m' and g^*. The equations are obtained by differentiation of (2a) w.r.t. g, (2b) w.r.t. g^*, and (7) w.r.t. m', respectively. Using $\alpha_5 := \alpha_2 + \alpha_4$ and $\beta_5 := \beta_2 + \beta_4$, we get:

$$(\alpha_1^2 + 1)g + \alpha_1\alpha_5 m' + \alpha_1\alpha_3 g^* = \alpha_1 \bar{y} \tag{2.1a}$$

$$\begin{aligned}[][w\alpha_1\alpha_5 \quad &+ \quad (1-w)\beta_1\beta_5]g + [w\alpha_5^2 + (1-w)\beta_5^2 + 1]m' + \\ &+ \quad [w\alpha_3\alpha_5 + (1-w)\beta_3\beta_5]g^* = w\alpha_5\bar{y} + (1-w)\beta_5\bar{y}^* \end{aligned} \tag{2.1b}$$

$$\beta_1\beta_3 g + \beta_3\beta_5 m' + (\beta_3^2 + 1)g^* = \beta_3\bar{y}^* \tag{2.1c}$$

Introducing

$$A' := \begin{pmatrix} \alpha_1^2 + 1 & \alpha_1\alpha_5 & \alpha_1\alpha_3 \\ w\alpha_1\alpha_5 + (1-w)\beta_1\beta_5 & w\alpha_5^2 + (1-w)\beta_5^2 + 1 & w\alpha_3\alpha_5 + (1-w)\beta_3\beta_5 \\ \beta_1\beta_3 & \beta_3\beta_5 & \beta_3^2 + 1 \end{pmatrix}$$

$$\underline{b}' := \begin{pmatrix} \alpha_1\bar{y} \\ w\alpha_5\bar{y} + (1-w)\beta_5\bar{y}^* \\ \beta_3\bar{y}^* \end{pmatrix} \text{ and } \underline{u}' := \begin{pmatrix} g \\ m' \\ g^* \end{pmatrix}$$

we obtain, similarly as in appendix 1, that a unique equilibrium strategy exists if and only if the determinant of A', denoted by $\det_2$, differs from zero.
Straightforward calculations show that

$$\begin{aligned} \det_2 \quad = \quad & (1 + \alpha_1^2 + w\alpha_5^2)(1 + \beta_3^2 + (1-w)\beta_5^2) - \\ & -(\alpha_1\beta_1 + w\alpha_5\beta_5)(\alpha_3\beta_3 + (1-w)\alpha_5\beta_5) \end{aligned}$$

Next we show that

$$u'^e := \frac{1}{\det_2} \begin{pmatrix} \alpha_1[(1 + (1-w)\beta_5^2 + \beta_3^2)\bar{y} - ((1-w)\alpha_5\beta_5 + \alpha_3\beta_3)\bar{y}^*] \\ (w\alpha_5(1 + \beta_3^2) - (1-w)\alpha_1\beta_1\beta_5)\bar{y} + ((1-w)(1 + \alpha_1^2)\beta_5 - w\alpha_5\alpha_3\beta_3)\bar{y}^* \\ \beta_3[(1 + w\alpha_5^2 + \alpha_1^2)\bar{y}^* - (w\alpha_5\beta_5 + \alpha_1\beta_1)\bar{y}] \end{pmatrix}$$

satisfies (2.1). This can be shown by direct substitution of u'^e into (2.1). We will only show the correctness of (2.1a) here. The other equalities can be shown similarly. We have:

$$
\begin{aligned}
&\det{}_2[(\alpha_1^2+1)g^e+\alpha_1\alpha_5 m'^e+\alpha_1\alpha_3 g^{*^e}] = \\
&\quad [(\alpha_1^2+1)\alpha_1(1+(1-w)\beta_5^2+\beta_3^2)+\alpha_1\alpha_5(w\alpha_5(1+\beta_3^2)-(1-w)\alpha_1\beta_1\beta_5) \\
&\quad -\alpha_1\alpha_3\beta_3(w\alpha_5\beta_5+\alpha_1\beta_1)]\bar{y}+ \\
&\quad [-(\alpha_1^2+1)\alpha_1((1-w)\alpha_5\beta_5+\alpha_3\beta_3)+\alpha_1\alpha_5((1-w)(1+\alpha_1^2)\beta_5-w\alpha_5\alpha_3\beta_3) \\
&\quad +\alpha_1\alpha_3\beta_3(1+w\alpha_5^2+\alpha_1^2)]\bar{y}^* = \\
&\quad \alpha_1[(\alpha_1^2+1)(1+(1-w)\beta_5^2+\beta_3^2)+w\alpha_5^2(1+\beta_3^2)+w\alpha_5^2(1-w)\beta_5^2 \\
&\quad -w\alpha_5^2(1-w)\beta_5^2-(1-w)\alpha_1\beta_1\alpha_5\beta_5-\alpha_3\beta_3(w\alpha_5\beta_5+\alpha_1\beta_1)]\bar{y} = \\
&\quad \alpha_1[(\alpha_1^2+1+w\alpha_5^2)(1+(1-w)\beta_5^2+\beta_3^2)- \\
&\quad (\alpha_1\beta_1+w\alpha_5\beta_5)(\alpha_3\beta_3+(1-w)\alpha_5\beta_5)]\bar{y} = \alpha_1\det{}_2\bar{y},
\end{aligned}
$$

which completes this part of the proof.
Next, consider the welfare-loss functions (2). Substitution of $\underline{u}'^e$ into (2a) gives:

$$
\begin{aligned}
J^e_{\mathrm{EMU}} &:= ((\alpha_1\ \alpha_5\ \alpha_3)\underline{u}'^e-\bar{y})^2+g^{e^2}+m'^{e^2} \\
&= \frac{1}{\det_2^2}\{((\alpha_1\ \alpha_5\ \alpha_3)\underline{u}'^e\det{}_2-\bar{y}\det{}_2)^2+(\det{}_2 g^e)^2\}+m'^{e^2}
\end{aligned}
$$

Now, $(\alpha_1\ \alpha_5\ \alpha_3)\underline{u}'^e\det_2-\bar{y}\det_2=$

$$
\begin{aligned}
&[\alpha_1^2(1+(1-w)\beta_5^2+\beta_3^2)+\alpha_5(w\alpha_5(1+\beta_3^2)-(1-w)\alpha_1\beta_1\beta_5) \\
&\quad -\alpha_3\beta_3(w\alpha_5\beta_5+\alpha_1\beta_1)-\det{}_2]\bar{y}+ \\
&\quad [-\alpha_1^2((1-w)\alpha_5\beta_5+\alpha_3\beta_3)+\alpha_5((1-w)(1+\alpha_1^2)\beta_5-w\alpha_5\alpha_3\beta_3)+ \\
&\quad +\alpha_3\beta_3(1+w\alpha_5^2+\alpha_1^2)]\bar{y}^* = \\
&\quad -[1+\beta_3^2+(1-w)\beta_5^2]\bar{y}+[(1-w)\alpha_5\beta_5+\alpha_3\beta_3]\bar{y}^*
\end{aligned}
$$

So, $J^e_{\mathrm{EMU}}=\frac{1}{\det_2^2}\{(1+\alpha_1^2)[(1+\beta_3^2+(1-w)\beta_5^2)\bar{y}-((1-w)\alpha_5\beta_5+\alpha_3\beta_3)\bar{y}^*]^2\}+m'^{e^2}$, from which immediately the result follows stated in (10a).

Note that, in case $\alpha_1 \neq 0$, also a similar proof of this property like at the end of appendix 1 can be given.

Appendix 3

In this appendix we calculate the equilibrium strategies and corresponding welfare-loss functions for both the non-EMU and EMU case in the three country model. Similarly as in the two previous appendices the both necessary and sufficient conditions for existence of an equilibrium strategy in the non-EMU case are obtained by differentiation of J, J^* and J_s w.r.t. g, m, g^*, m^*, g_s and m_s, respectively. This yields the four equations (1.1) presented in appendix 1 together with the equations:

$$\gamma_1\gamma_5 g + \gamma_2\gamma_5 m + \gamma_3\gamma_5 g^* + \gamma_4\gamma_5 m^* + (\gamma_5^2 + 1)g_s + \gamma_6\gamma_5 m_s = \gamma_5 \bar{y}_s \tag{2.1}$$

and

$$\gamma_1\gamma_6 g + \gamma_2\gamma_6 m + \gamma_3\gamma_6 g^* + \gamma_4\gamma_6 m^* + \gamma_5\gamma_6 g_s + (\gamma_6^2 + 1)m_s = \gamma_6 \bar{y}_s \tag{2.2}$$

As already shown in appendix 1 (though in a somewhat more general context), (18a-b) satisfy the four equations (1.1).

We will next show that (18a-d) also satisfy (i). That (ii) is also satisfied by these expressions can be shown in a similar way.
Using, the relationships $g^e = g^{*^e}$, $m^e = m^{*^e}$ and $\gamma_6 g_s^e = \gamma_5 m_s^e$, we have that

$$\begin{aligned}
& d \cdot \det \cdot [(\gamma_1 + \gamma_3)\gamma_5 g^e + (\gamma_2 + \gamma_4)\gamma_5 m^e + (1 + \gamma_5^2 + \gamma_6^2)g_s^e] = \\
& \quad (\alpha_1(\gamma_1 + \gamma_3)\gamma_5 + \alpha_2(\gamma_2 + \gamma_4)\gamma_5) d(1 + (1 - \theta_1)\alpha_1^2 + (1 - \theta_2)\alpha_2^2)\bar{y} + \\
& \quad +(1 + \gamma_5^2 + \gamma_6^2)\gamma_5 \det[-((\gamma_1 + \gamma_3)\alpha_1 + (\gamma_2 + \gamma_4)\alpha_2)\bar{y} + \\
& \quad +(1 + (1 + \theta_1)\alpha_1^2 + (1 + \theta_2)\alpha_2^2)\bar{y}_s] = \\
& \quad (d(1 + (1 - \theta_1)\alpha_1^2 + (1 - \theta_2)\alpha_2^2) - (1 + \gamma_5^2 + \gamma_6^2) \det) \\
& \quad (\alpha_1(\gamma_1 + \gamma_3)\gamma_5 + \alpha_2(\gamma_2 + \gamma_4)\gamma_5)\bar{y} + \\
& \quad (1 + (1 + \theta_1)\alpha_1^2 + (1 + \theta_2)\alpha_2^2)(1 + \gamma_5^2 + \gamma_6^2) \det \gamma_5 \bar{y}_s = \\
& \quad ((1 + (1 + \theta_1)\alpha_1^2 + (1 + \theta_2)\alpha_2^2)(1 + (1 - \theta_1)\alpha_1^2 + (1 - \theta_2)\alpha_2^2) - \det) \\
& \quad (1 + \gamma_5^2 + \gamma_6^2)(\alpha_1(\gamma_1 + \gamma_3)\gamma_5 + \alpha_2(\gamma_2 + \gamma_4)\gamma_5)\bar{y} + d \cdot \det \cdot \gamma_5 \bar{y}_s.
\end{aligned}$$

It is easily verified that $\det = (1 + (1 + \theta_1)\alpha_1^2 + (1 + \theta_2)\alpha_2^2)(1 + (1 - \theta_1)\alpha_1^2 + (1 - \theta_2)\alpha_2^2)$. Using this in the above expression it follows then straightforwardly that (18a-d) satisfy

(i). The correctness of the expression for the welfare-loss functions J and J^* (19a), follows directly from (13).
The correctness of (19b) can be proved analogous the proof for J^e we gave at the end of appendix 1.

Next, consider the EMU case.
Differentiation of J, J_{EMU}, J^* and J_s w.r.t. g, m', g^* and g_s, respectively, yields the equations:

$$\begin{aligned}
&(1+\alpha_1^2)g + \alpha_1\alpha_5 m' + \alpha_1^2\theta_1 g^* = \alpha_1\bar{y}\\
&(\alpha_1\alpha_5 w_1 + \alpha_1\alpha_5\theta_1 w_2 + \gamma_1\gamma_7 w_3)g + (1+(1-w_3)\alpha_5^2 + w_3\gamma_7^2)m' +\\
&\quad +(\alpha_1\alpha_5\theta_1 w_1 + \alpha_1\alpha_5 w_2 + \gamma_3\gamma_7 w_3)g^* + w_3\gamma_5\gamma_7 g_s = (1-w_3)\alpha_5\bar{y} + w_3\gamma_7\bar{y}_s\\
&\alpha_1^2\theta_1 g + \alpha_1\alpha_5 m' + (1+\alpha_1^2)g^* = \alpha_1\bar{y}\\
&\gamma_1\gamma_5 g + \gamma_1\gamma_7 m' + \gamma_1\gamma_3 g^* + (1+\gamma_5^2)g_s = \gamma_5\bar{y}_s
\end{aligned}$$

Substitution of (20) shows that these variables satisfy the equations. In other words, these are the equilibrium strategies.
A similar reasoning like at the end of appendix 1 shows then

$$J^e_{\text{EMU}} = J^{*^e}_{\text{EMU}} = \left(\frac{1}{\alpha_1^2}+1\right) g^{e^2}_{\text{EMU},3} + m'^{e^2}_{\text{EMU}}$$

and

$$J^e_{s,\text{EMU}} = \left(\frac{1}{\gamma_5^2}+1\right) g^{e^2}_{s,\text{EMU},3} + m'^{e^2}_{\text{EMU}}$$